Talk that was not given. 1/6 Scan#8

LONDON.

I very much appreciate your asking me to become your president and to give this talk. I understand from your secretary that you have had your fill of thinking about the National Curriculum and would like this talk to have a substantial mathematical content. I shall certainly not talk about the National Curriculum, about which I know practically nothing. However I have been involved for so long in the effort to spread good mathematics teaching that I doubt if I am now capable of giving a talk that does not have some relevance to that struggle. What I plan to do is to give a rather meandering talk about various things I have run across in the last 70 years, and an appreciable amount of mathematical information will come into this.

SACRAMENT.

At one time, among other duties, I took part in the post-graduate training of mathematics teachers. I was struck by the attitude of some of my colleagues. They seemed to regard this course as a kind of sacramental laying-on of hands without which no one could be a good teacher. This did not strike me as very realistic. It seemed to me that many of the attitudes and ideas that would make a good or a bad teacher were absorbed much earlier in life. I remember conclusions I reached from observing my form master when I was eleven years old.

He was a tall, gaunt man with a slightly unusual background, being a Canon in the Church of England and having at one time been in the Canadian Mounted Police. I remember him quoting something which I later discovered was from Ian Hay's **The Lighter Side of School Life.** "In nine cases out of ten a schoolmaster's task is not to bring light to the path of an eager, groping disciple, but to drag a reluctant and refractory young animal up the slopes of Parnassus by the scruff of his neck." The implication seemed to be that we had a moral duty to work hard and that we were neglecting this. Whether children have such a duty is open to question, but one thing is quite certain - they do not feel they have. It is futile appealing to feelings pupils ought to have but do not.

Again, still mourning our deficiencies, he used to say, "You can take a horse to the water but you cannot make him drink". At the time I thought, perhaps you could give him something that would make him thirsty.

It is important to realize that some parts of the mind's action are automatic. You cannot become interested by an act of will. Being interested, like falling in love or laughter, is something that happens to you. It is no use my saying, "I will tell you a joke and I want you to try hard to laugh at it".

If one stimulus does not work, you have to search for another that does, and in this you have to be supremely realistic, not confusing what would suit you with what actually exists.

SWEARING AT VICE-PRINCIPAL.

Sometimes in seeking for such a stimulus you have to be a little unorthodox. I was once told about a girl in a London school who was causing great trouble, including swearing at the Vice-Principal. I asked what she was interested in, and was rather surprised to be told she had a passion for the French language. I said that, if I had her in a mathematics class, I would begin by simply letting her work at French; this might give her the feeling I was on her side. Later I would show her some French publication containing mathematics - perhaps puzzles - and suggest she translate it for the benefit of the rest of the class. How things went after that we would have to see. I do not know if my advice was taken.

This has some interest for me in that my own feeling about subjects had been exactly the reverse. I went to a rather unusual school in Sunderland, where we studied all the academic subjects at full force from the age of eight on. Mathematics was already a subject of immense appeal for me; French I did badly at and saw no purpose in. If I had had a teacher of genius who had told me about, or even better given me, a book on mathematics in French that was not available in English, my whole attitude to the subject would have been reversed. It was not that I consciously tried to neglect French; it was rather that the deep drive from the unconscious was lacking.

KING SIDE ATTACK.

It is a maxim in chess that, if you are launching an attack on the king's side, you must throw everything you have into it. A similar maxim applies in using an unorthodox approach. You must be willing to do whatever the logic of the situation requires. Of course, you need to look for a way of doing this that does not involve your getting the sack.

HISTORIAN STUDENT.

I once had to deal with a situation where a young man was in the class who should not have been there at all. I had been asked to go into a school and teach calculus to a class roughly corresponding to a sixth form here. One of the students was an outstanding historian; his father, who was a brilliant but stupid scientist, insisted he take every possible mathematics class. He had no interest whatever in calculus. I could have gone through the motions of pretending to teach it to him, but this would have been a waste of his life and mine. Instead I mentioned a question that arises for each subject; is its history self-contained or does it reflect the development of society? For instance in music is it sufficient to compare Beethoven with Haydn and Mozart, or do you have to bring in the French Revolution and the rise of romantic nationalism?

MARX

I talked to him about people who have discussed this in relation to mathematics. One of them was Karl Marx, who had a balanced view on this question. According to him, mathematics develops in two ways, partly through its own internal logic and partly through the pressure of outside events.

An example of internal development, I suppose, would be Gauss' discovery that the regular figure with 17 sides could be constructed with ruler and compass. The Greeks had succeeded in constructing the regular pentagon - which in itself is a remarkable achievement. Gauss evidently wondered what property of 5 made this possible and proved that 17 had a similar property. Now the construction of a regular

17-sided figure never has had any economic purpose, and I imagine it never will.

On the other hand in the 17th century, for instance, mathematics was intimately tied to the social needs of the time. One of these was for accurate navigation across the Atlantic. The documents of the Royal Society open with an address to the future James II, then Lord High Admiral. It contains the passage;-

"All **Art** and **Nature** will exert their powers to keep pace with **Astronomy**; particularly **Navigation** being under your **Royal Highness's** Immediate **Care)** will industriously apply these **Accurate Observations** to all **Nautical** Purposes, and by some Familiar Method, deliver the **Anxious Seamen** from the **Fatal Accidents** that frequently attend their **Mistaken Longitude.**"

They had some reason to be anxious. Years ago I read a statement by some Elizabethan mariners, to the effect that they sailed past island A believing it to be island B. The islands in question were 200 miles apart.

Newton's Principia goes into immense detail on the motion of the moon. You might have thought that he would have had sufficient weighty material, just showing that the inverse square law of gravitation accounted for the motion of the planets, without going into minute details of what the moon did. The explanation is related to navigation. To-day a captain can determine his longitude by comparing local time with the radio signal for Greenwich time. In the 17th century the sky was the sailor's clock. If you knew exactly where the moon should be at any given time, you could find the time by observing the sky, so the lives of men depended on Newton's calculations.

Many other social aspects of Newton's work can be found in a book by a Russian writer, who was later murdered by Stalin.*

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*B.Hessen. Social and Economic Roots of Newton's Principia.

PETER THE GREAT

The English concern with practical mathematics had interesting results around 1700, when Peter the Great of Russia visited England . Peter wanted Russia to acquire a navy. In order to learn as much as possible about ships, he worked as a shipwright, first in Holland and then here in London, at Deptford. He was disappointed in the Dutch. For them shipbuilding was a craft, handed down from father to son. Peter said all he learnt from them was some carpentry. In England ships were designed on mathematical and scientific principles. He decided that all the men he invited to Russia to supervise the building of his navy should be English.

SPENGLER

The other man I mentioned to my student was Spengler. Spengler was an incredibly learned German schoolteacher, who published a book that aroused widespread interest in the years following the first World War. According to him there was an analogy between the history of a culture and the life of an individual. A culture was born. It was destined to have a vigorous youth, a productive maturity and finally a stage of decadence in which traditional beliefs and loyalties faded and money dominated everything. All human activities - art, music, literature, architecture, philosophy, mathematics, science and so forth - reflected both the stage the culture had reached and the individual style of the culture.

I told my student that his work for the term would be to write a paper on the relation of mathematics to history.

I believe that what I did was correct, not only for the needs of the student in question, but also for spreading mathematical knowledge. I am sure he learned far more about mathematics this way than if I had concentrated on how to differentiate e^x sinx.

BRUTUS

One finds various interesting pieces of information in Spengler. I did **Julius Caesar** more than once at school, but I never realized that Brutus was an extortioner millionaire who murdered the democratic Caesar with the approval of a reactionary Senate.

AN INCORRECT PROPHESY.

Mathematics provides us with a proof that Spengler was not correct on all counts . Spengler was very interested in mathematics and perhaps as well-informed as many mathematicians of his day. At the end of his chapter on mathematics he says that for us the time of great, creative mathematics is past; we can only do "clever detail-work" . He could not have been more wrong. Already in 1900-1910, there had occurred three revolutionary innovations in mathematics, which many mathematicians themselves did not appreciate until years later. These were Hilbert space, Lebesgue integration, and Frechet abstract spaces.

Far from concentrating on details, modern mathematicians tend to be criticized for losing themselves in vast generalizations and not bothering about particular problems.

MODERN MATHS; MISCONCEPTIONS.

A few years ago, mathematicians caused considerable turbulence in education by insisting that teaching in secondary, and even in primary schools, should be modified to take account of their discoveries. One misconception, prevalent at that time, was that modern mathematics is totally unconnected with traditional mathematics, and makes the older mathematics obsolete. This is absolute nonsense as we can see from the example of Frechet's work. One thing which led to Frechet's work was the 19th century study of the calculus of variations,in which the unknown is not a number but a function. Suppose, for example, we are studying a soap bubble stretched between two circular wires. The shape of the soap bubble will be given by the surface of rotation that has the minimum area. The problem is to find the function that gives this.

Hadamard pointed out that, when we are dealing with real or complex numbers, we can visualize them sitting on a line or a plane. Analysis is much concerned with limits. Any statement about limits can be visualized in terms of distances. If a point is moving towards a limit, its distance from the limit point is tending to zero. We do not have any corresponding picture when a sequence of functions are tending to a limit. Frechet set out to remove this difference. He showed that it was possible to define distance for all kinds of things other than numbers - functions, matrices, operations.

The guiding principle in Frechet's work was to look for theorems resembling those already known for real and complex variable. This meant that the new mathematics was related to the old in two ways. It arose as a generalization of the old, and it provided powerful new weapons for attacking traditional problems - amd even to-day many problems about the real world are expressed in the traditional symbols.

A SIMPLE METRIC SPACE.

When the distances between mathematical objects have been suitably defined, the result is called **a metric space.**

As a simple example of a metric space we may consider 2 x 2 matrices. We define the distance between two matrices as follows; suppose the first matrix sends some vector to **OP** while the second sends it to **OQ.** The distance between the two matrices is defined as the maximum value of the distance PQ **that can occur for a vector of unit length.** This definition serves to define distances between operations that can be represented by matrices, such as rotations about the origin or reflections in lines through the origin.

In very simple cases we can show the metric space by an actual drawing.

It is not hard to check that the distance between rotations through (say) 37° and 53° is the same as the distance between points on the unit circle at these same angles, 37° and 53°, so we can take each point on the unit circle as the place for the corresponding rotation about the origin.

It is also possible to associate reflections about lines through the origin with points of the unit circle in such a way that the distances between these points accurately represent the distances between the reflections.

Finally, we may consider the distance between a rotation and a reflection. It turns out that this distance is always 2, whatever the rotation and reflection considered. That all these distances are the same is not surprising. It would be surprising if it turned out that some reflections were more like a rotation than others.

We can get a picture of the metric space as follows. Suppose rotations to correspond to points of the unit circle marked on the paper, and reflections to points marked on the same circle, but on the underside of the paper. The only way of getting from one circle to the other is through a pinprick at the origin!

Such a physical model is exceptional. We cannot, as a rule, see with our physical eye the geometry corresponding to these distances, but geometrical language suggests very fruitful analogies. The power of the new method is immense, since it can be applied to such a wide range of mathematical objects.

THE PROPER ROLE OF MODERN MATHS.

In the 1960s mathematicians were insisting that schools should be influenced by the discoveries of this century, and I would like to consider how far this view is sound. It should be emphasized that the situation in America was very different from that here. In this country the SMP books, for instance, were written by very experienced and skilful schoolteachers. (Admittedly, they made some mistakes, for instance, the a reaction against excessive drilling in algebra lessons led to the complete abandonment of algebraic manipulation.) In the USA however, while the movement involved some mathematicians who were good teachers, the decisive role was taken by mathematicians who had no experience of school teaching whatever. The result, as one might expect, was a disaster and this is generally accepted in the United States to-day.

My first impression when I went to America was simply that the Americans had gone mad. After some time, I managed to understand how things had developed. It began, as such movements usually do, with a perfectly justifiable complaint. The graduate schools found that undergraduates came to them with a sound enough understanding of 18th and 19th century mathematics, but in no way equipped to cope with the research of the present century. They put forward some ideas which were perfectly reasonable for the teaching **at undergraduate level of students who were going on to do mathematical research.**

Then the Russians put Sputnik up and there was panic that the Unite States was falling behind because its mathematics was not good enough. The government put millions of dollars at the disposal of the mathematicians, who were told to put this right.
The mathematicians were delighted, but it did not occur to them that a strategy, designed for future research workers, might not be just what was needed by the entire nation. An added irony was that American mathematicians were extremely pure and despised anyone who was concerned with applying mathematics to physics or engineering.

LESSONS FOR BRITAIN.

What was the sound element in this movement and what implications are there for this country?

In America there was a marked difference between graduate and undergraduate mathematics. The thing here that most closely parallels that gap is the difference between the way things are handled in universities and in Sixth Forms. The universities are making some efforts to ease this transition, but the main responsibility for preparing students to cope with university mathematics, I believe, rests with the teachers in schools.

In a university lecture you can be sure all the appropriate results will be stated and proved. But students are not always put in a position to see what the whole course is trying to do, where it came from and where it is going. I remember when I was at Cambridge I heard of only two lecturers who discussed the history of the subject.

ANALYSIS.

I would like to illustrate this point by discussing analysis. I believe there was a survey which showed that at the part of mathematics pupils enjoyed most at school was calculus, and at the university what students enjoyed least was analysis. In school, calculus is intuitive; you accept something if it sounds reasonable and looks right. In universities it is exactly the opposite; everything must be proved with the utmost, legalistic precision.

The schools approach corresponds to the way mathematicians worked in the 17th and 18th centuries, the universities to the way mathematicians thought in the 19th century.

Now in fact there is a very interesting explanation why, at a certain stage of history, mathematics changed from the first approach to the second. It arose from the interaction of music and mathematics. I sketched how this came about in an article in the sixth-form magazine, **Mathematical Review.**

In 1715 Brook Taylor showed that the vibrations of a string corresponded to formulas such as

$y = \sin \mathbf{x} \cos \mathbf{t}$	Fundamental.
$y = \sin \mathbf{2x} \cos \mathbf{2t}$	First Harmonic.
$y = \sin \mathbf{3x} \cos \mathbf{3t}$	Second Harmonic, and so on.

The French composer Rameau said that he heard many of these harmonics at one and the same time when an instrument was played, and this led Daniel Bernouilli to consider

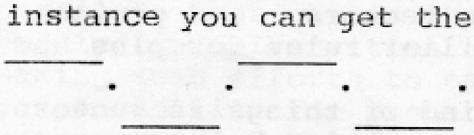
$$y = c_1 \sin x \cos t + c_2 \sin 2x \cos 2t + \ldots \quad (I)$$

as the formula for the general vibration of a string. At time $t=0$ this gives the shape of the string as

$$y = c_1 \sin x + c_2 \sin 2x + c_3 \sin 3x + \ldots (II)$$

There are an infinity of constants in each of these, and Bernouilli maintained that they covered all possible vibrations. Euler and d'Alembert disagreed strongly. They said the simplest way to set a string in vibration was to pull the mid-point aside so the string had the shape /\
/ \ They did not believe that a single formula could give parts of two different lines. Mathematicians argued for at least half a century without coming to agreement.

In fact series such as the above can do far worse things. For instance you can get the graph



in which there are actual breaks, although each of the sine functions is continuous. Indeed, it is possible to get graphs that cannot be drawn because there are infinitely many breaks between any two points on them. Thus monsters arise that had never been imagined before, and it is not surprising that much more careful logic is needed to deal with them.

It would make lectures on analysis much more comprehensible if they began with some such sketch of the historical background.

It is perfectly possible to bring such things to the attention of Sixth-Formers by means of exercises perfectly relevant to A-level work. In chapter 4 of Piaggio's **Differential Equations** there will be found exercises that are quite suitable for practice in integration, and lead, for example, to the graph in the previous paragraph. They also have something to do with a piece of modern maths, Hilbert space.

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Suppose, for instance, we want to find a series of the type (II) that will make y = 1 for 0 < x < n. We are helped by the following formulas, which are easily verified at A-level.

(a) $_0^n$ sin mx sin nx dx = 0 if m , n are different whole numbers.

(b) $_0^n$ $\sin^2$ nx = n / 2.

If we want to find, say, c_3 we take the equation

$$1 = c_1 \sin x + c_2 \sin 2x + c_3 \sin 3x + \ldots\ldots$$

multiply it by sin 3x and integrate from 0 to n. (a) shows that all the constants except c_3 disappear, and (b) shows that we have

$$(n / 2)\ c_3 = \ _0^n \sin 3x\ dx\ .$$

(This step requires justification. In general you are not allowed to integrate an infinite series term by term.)

In the same way we can find the other constants.

Now the surprises begin. First of all, it is clear that, if x=0, we have y=0. Again, changing x to -x makes y change to -y, so y= -1 for x between - n and 0. The graph keeps repeating the part from - n to n , and we get something that looks like the battlements of an ancient castle.

All of this was known, to some mathematicians at any rate, as early as the 18th century.

HILBERT SPACE.

It was only in the present century that a remarkable geometrical interpretation was found.

One feature of modern maths must be understood. In classical mathematics a concept stood for something definite ; a vector for instance was something that could be drawn as an arrow. The modern approach is abstract and general; it asks,"What properties do vectors have that make them convenient for calculation?" The essential properties are very slight - you can add two vectors and you can multiply a vector by a constant, and the familiar rules for **plus** and **times** apply.

Thus it becomes possible to talk of all kind of things as vectors; in particular, functions qualify as vectors; we know what we mean by f(x) + g(x) and kf(x).

In some circumstances we can find a suitable generalization of the dot product. For functions we may take the integral of f(x)g(x) over an agreed interval. If this is zero, we may say the functions are perpendicular. In the literature this is usually called **orthogonal.**

If we take (0, n) as the interval, equation (a) above is concerned with a dot product; this being zero, sin mx and sin nx are orthogonal when m and n are different whole numbers.

Thus sin x, sin 2x, sin 3x,.... form a system of perpendicular vectors. We could regard these vectors as being the axes of a co-ordinate system. The component of a vector along a particular axis is usually found by taking the dot product of the vector and a vector along that axis. **This is precisely what we did to find c_3 earlier.** Thus - although this was not realized for 150 years - the procedure used to find the coefficient in a series of sines is related to an extremely familiar one.

ORTHOGONAL SYSTEMS.

A set of mutually perpendicular vectors is known as an orthogonal system. We have seen that the sines are a particular example of this, but it is clear they are only one in a great multitude of such systems.

We can see this in two ways. The first is purely geometrical - we have an immense choice when we are selecting a co-ordinate system with perpendicular axes.

The second way is connected with mechanics. I find difficulty in stating this precisely but I have a strong feeling that the integral being zero is related to the fact that the forces arising in one of the harmonics do not tend to produce another harmonic. Whether this is so or not, the fact is that orthogonal functions appear in oscillations of many kinds - mechanical, electrical, atomic. They are thus a central topic in physics.

LEGENDRE POLYNOMIALS.

It is possible to have an orthogonal system more elementary than the sines. The functions can be polynomials. If we take our interval as -1 to +1, the following polynomials will do; -

$1, x, 3x^2 - 1, 5x^3 - 3x, 35x^4 - 30x^2 + 3, \ldots\ldots$

Someone just beginning integration can check that, if you multiply two of these together and integrate from -1 to +1, you get 0.

To get the polynomial of degree n, you work out

$(d/dx)^n (x^2 - 1)^n$ and then multiply or divide by any constant you may fancy.

These are known as Legendre Polynomials and play a great role in the theory of electric fields. See, for instance, Jeans, Theory of Electricity and Magnetism.

A CONJURING TRICK.

I am now turning to something more elementary. Conjuring tricks are useful; if you are doing something that seems quite impossible, this arouses a genuine desire to know what is happening. Over the years I have worked out a certain routine; it can be done with a class, of course, and I have done it successfully on several occasions with an audience of 400 Sixth Formers. Birmingham once prevailed on me, against my better judgement, to try it for an audience of 1100 with elaborate technology, such as portable microphones. It was a complete shambles.

I described it as a method of discovery in **Search For Pattern** and since then have added a new item to it, called **Desert Island Trig Tables.**

The trick begins by asking each member of the audience to choose a quadratic expression, and tell me its values for x = 0,1,2,3,4. Someone might give me, say, 11,7,9,17,31. Sometimes there is a slip in the arithmetic, so I begin by checking the accuracy, using a method I first met during the war, when I was a very junior member of a team working on the design of radar. Incidentally, it impresses people very much if you can tell them there is a mistake in their data before you have found what formula they are using. The check is simply to make a table of differences.

```
 11     7     9    17    31
    -4     2     8    14
        6     6     6.
```

The numbers in the bottom row are all the same, there is no mistake.

I immediately say, "Your formula is $3x^2 -7x + 11$". The blackboards are then covered with such tables. I announce the formulas behind them, and the audience try to guess how it is done. It is not long before people notice that 11 is the first number in the top row and the coefficient of x^2 is half the number in the bottom row. It takes longer for them to see that the coefficient of x, -7, comes by subtracting the 3, that goes with x^2, from the -4 at the beginning of the middle row.

Various problems and puzzles are then solved by using this result and its correctness is checked by considering the general quadratic.

The Desert Island problem is now this. You have made a boat and instruments of navigation in order to escape from the island. You now need to use what you remember of school trigonometry to make a table of sines. The solution uses the idea that, if different people draw a smooth curve through three different points, their curves will usually not differ by much. Suppose we plot the points (0, sin 30°), (1, sin 45°) and (2, sin 60°) and fit a quadratic to these. We then have the table

```
0.5        0.707        0.866
     0.207        0.159
          -0.048
```

Our rule gives $y = -0.024\, x^2 + 0.231\, x + 0.5$. We find, for example, sin 35°. This corresponds to x = 1/3. Our formula gives 0.57433. The accurate value is 0.57358, with a difference of 0.00076 - probably much less than the errors introduced by using a sextant made from palm wood.

In fact trig tables have been made using essentially this idea. A number of values are calculated very exactly, and sufficiently close together. The intermediate values are found by fitting a polynomial. The difference table allows us to find the values using addition only, which is a much quicker operation, even for a computer, than multiplication.

This trick is in fact very close to methods important for modern numerical analysis. Even before the age of computers, a way of linking adding machines together was devised, so that tables were printed automatically without the possibility of human error.

THE MATHEMATICAL TRADITION.

In conclusion, I want to return to a question I mentioned at the beginning, that of preserving and spreading good teaching.

One reason, not always mentioned, for wanting good teaching of mathematics is given by something said to me many years ago by Dr. Margaret Lowenfeld. She ran an excellent clinic in London for children who had emotional difficulties. She told me they found it necessary give the children some hints on arithmetic. Otherwise it was likely that, when they returned to school, the anxiety created by arithmetic lessons would undo the good the clinic's treatment had done.

A good young teacher I knew told me he learned two useful things in his post-graduate year in Toronto. One was a discovery procedure we used in our maths classes; the other was Plato's **Meno** in which Socrates, by simply asking questions, gets a slave boy to arrive at a particular case of Pythagoras Theorem. The point I am making is that throughout the centuries there have been people who understood how to teach mathematics. Sometimes there have been many, sometimes few. Our aim is that this understanding should spread out from them to wider circles.

What is required is a combination of sufficient mathematical knowledge and the ability to teach. In the early days of our Association something happened that has not occurred anywhere else. In the 19th century, and in part of the 20th, many of the ablest mathematicians on graduating preferred to teach in private schools rather than to take junior posts in universities. Thus many of the teachers who fought for reform in the 19th century had better mathematical qualifications than the hidebound professors opposing them. Naturally, as sensitive and intelligent people in direct contact with school pupils, they soon acquired the art of teaching well.

In the Ministry of Education pamphlet No. 36, **The teaching of Mathematics in Secondary Schools** you will find a summary of the Association's 1919 Report. In about 5 pages you will find almost every principle of good mathematics teaching set out. I was fortunate to be born at the right time to benefit when these recommendations were first put into practice.

THE PRESENT SITUATION.

To-day of course things are far different. New forms of employment for mathematicians have come into existence and there is a desperate shortage of well qualified mathematics teachers. I am not necessarily speaking of advanced mathematics.

PRIMARY SCHOOLS.

A high-school teacher I knew in America asked his students, "Do you like or dislike mathematics, and which teacher made you feel that way?" Nobody mentioned a secondary school teacher. Whether they were for mathematics or against it, the feeling had been created in primary school.

The qualifications for teaching mathematics in primary school are (1) to enjoy mathematics, (2) to expect the pupils to enjoy mathematics, (3) to see mathematics as something you can discuss and arrive at conclusions, (4) knowledge adequate to the tasks in hand.

HELP LINES.

A teacher who is not a mathematical specialist may well hesitate to initiate a discovery lesson in case it leads to questions he or she cannot answer. There are two things to be said here; first, there are properties of numbers that a 9-year-old can understand and that mathematicians have been trying unsuccessfully to explain for centuries, so there is no need to feel ashamed if something comes up that you cannot deal with. Secondly, there should be some regular procedure for dealing with a problem beyond the powers of a teacher. The local branch of the Mathematical Association might make itself available for consultation, or some bright pupil in a nearby secondary school might be glad to work on the question.

This brings me back to something I touched on earlier - teacher training begins long before you get anywhere near teacher training. The Americans have a very sound idea; within schools they have clubs called **Future Teachers of America.** Any way in which you can give a pupil opportunity to help weaker pupils is an excellent way of giving valuable early experience.

ABLER PUPILS.

For the last 14 years I have been having 4 or 5 particularly able ai interested secondary school pupils come to our home for a couple of hours on Saturday mornings. This of course is on a minute scale, but is yet significant. Two of the students I had in the early days have since done significant research, one in veterinary science, the other in computer interpretation of genetics data. It is the brightest students who suffer most from the shortage of top-flight mathematicians in the schools and from the lock-step organization of many classes. One important aspect of our Saturday work is that pupils are often at different stages, and so they have to read for themselves while I talk to one or two of them. Reading is the only hope of the able. There is no curriculum that could conceivably cover what a good young mathematician is capable of learning while still at school.

THE ROLE OF LEADERS.

I was at university at the time of the great depression and the rise of the dictators. Everyone was very political and discussing what ought to be done. That of course left the question of how to get it done. I decided to read the life of Wilberforce and see how things worked out in the movement to abolish slavery. One thing struck me very forcibly. Immediately after Wilberforce made his first speech in the House of Commons, letters applauding him came from all parts of the country. In fact the movement was there before the leader, and this I believe is generally the case. A leader is like a burning glass concentrating the rays of the sun. If the sun is not shining, nothing dramatic is going to happen.

The situation at present is neither excellent nor hopeless. There are many unfavourable factors, but there are also many people who have the correct feeling for mathematics. We should work on whatever scale the situation permits. If we help the development of even one person who may in the future, as a teacher, parent or employer become a centre from which good understanding of mathematics can spread out, that is something well worth doing.