Extract from

"A STRATEGY FOR IMPROVING EDUCATION AT ALL LEVELS, WITH PARTICULAR REFERENCE TO MATHEMATICS"

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The last thing I want to do is to start some new bandwagon movement and more violent oscillations in educational practice. I would therefore emphasise strongly that, as I see it, when you are trying to improve education, you have to be prepared to go through three stages. In the first stage, you have an idea that some improvement is possible, but you are groping in the dark as to how to bring it about. In the second stage you are learning a new technique of teaching, you are acquiring the art of bringing about the situation you believe possible. This is often a long and painful process, involving many false starts and mistakes. Thirdly, you try to persuade other teachers to do something similar, in the hope that gradually a tradition of good teaching will spread out and become established as a natural part of general practice.

What then starts us thinking that there may be some gap between schooling - what actually happens in schools - and education - what we would wish to achieve?

I would like to read you a quotation from H.G.Wells' History of Mr.Polly. H.G.Wells was born in 1866 and Mr.Polly was published in 1910. The quotation reflects some aspects of English education between those years.

> ... his mind was in much the same state that you would be in, dear reader, if you were operated upon for appendicitis by a well-meaning, bodly enterprising but rather overworked and underpaid butcher boy ... He thought of the present world no longer as a wonderland of experiences, but as geography and history, as the repeating of names that were hard to pronounce, and lists of products and populations and heights and lengths, and as lists and dates - oh! and Boredom indescribable .... he was always doubtful whether it was eight sevens or nine eights that was sixty-three (he knew no method for settling the difficulty) and he thought the merit of a drawing consisted in the care with which it was 'lined in'. 'Lined in' bored him beyond measure ...... Outside the regions devastated by the school curriculum he was still intensely curious.

H.G.Wells' verdict is pretty clear; the boy has gone to school eager to learn; the schools have destroyed that eagerness in everything they have touched.

In this connection one might mention the old distinction between "Teaching Subjects" and "Teaching Children". This is a totally unreal distinction, as you can see from the quotation above. Surely H.G.Wells is concerned with the child, and with what the schools have done to him. But the damage is due to the ideas the schools have about the subjects. They think drawing is concerned with painstaking shading-in. If they had understood what the subject of drawing really was, and had been able to communicate this, Mr.Polly would have revelled in it, and the same is true for history and literature and mathematics.

There is a very good description of what it feels like to love a subject in Ian Hunter's book on Memory[3].

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[3] Memory, by I.M.L.Hunter. (Penguin, 1968), page 123

> The interest of the dedicated scholar in learning about his subject derives from his concern for the subject itself, his genuine curiosity about the issues involved, his need to resolve perplexities; so strong is his interest in the subject that he memorises a great deal in an 'incidental' way, that is, without explicitly intending to memorise; his actively working with and thinking about the material is sufficient for its memorising without his having to set himself the task of learning it.

Hunter also quotes[4] the famous American psychologist

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[4] Memory, p.95

William James,-

> Most men have a good memory for facts connected with their own pursuits. The college athlete who remains a dunce at his books will astonish you by his knowledge of men's 'records' in various feats and games, and will be a walking dictionary of sporting statistics. The reason is that he is constantly going over these things in his mind, and comparing and making series of them.

These two passages, put side by side, bring out a rather striking fact - that the distinguished scholar and the keen sports fan, who outwardly appear so different, are in fact using their brains in exactly the same way.

I have for a long time believed that the thought processes of very young children closely resembled the thought processes of genius, and I was pleased recently when I found the same view expressed in the writings of Ostwald. Ostwald is important for us because he was not only a great scientist but also a very successful teacher. A very interesting chapter in his autobiography[5] tells how a Japanese student

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[5] Ostwald, Lebenslinien, Volume 3, chapter 4

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approached him and asked how one could spot able students very early. Ostwald asked why he wanted to know this, and was told that the Japanese government had established a scholarship scheme to raise the scientific level of their country, and wanted to make sure they aided the most promising youngsters. Ostwald had been approached because so many of his students had become distinguished scientists. Ostwald felt he did not do anything special, but decided to look into the question. He found that it was true that his students had done exceptionally well; many of them held leading positions, while students from another, much larger institution, were nearly all on the lowest rungs. He compared the methods used. In the other institution the students regarded the director with hero worship. If the director held a certain belief, the students were told to do experiments to prove it was right. If the director disliked the views of some rival scientist, the students did experiments to prove those views wrong. If the experiments did not lead to the desired conclusion, the students felt they had failed.

Ostwald himself worked in a very informal manner. The students' research had to be in a certain broad field of chemistry, but within that field they were left to choose a question that intrigued them and work on it. Ostwald told them that if theory predicted a certain result, and they verified by experiment that it was so, this was certainly a success. But it was a small success. The big success came when an experiment showed exactly the opposite of what was expected. In clearing up such a contradiction a whole new branch of science could result.

Ostwald thought about the question of how you could spot a young genius; he concluded that you should look for students who were thoroughly dissatisfied with school and felt cramped by it. The schools in question were of course of the rigid Prussian classical gymnasium type, but some echo of his remarks may perhaps be heard in the schools of today. The genius, he said, did not get his information from teachers, but from books, or from some older friend outside the school. The teachers did not think well of the genius; he was troublesome, he did not fit in, he asked questions they could not answer, he concentrated passionately on what he thought important and refused to waste time on other things the school demanded; he was extremely restless in a class that proceeded by the usual lock-step method with all the students working at the same pace. Ostwald went on to make studies of the lives of geniuses. In his book[6] on great men, he wrote,

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[6] Ostwald, Gross Manner, p.45

> When the qualities of great men become better understood, the conditions favourable to their development will be more surely recognised and established. And as the genius differs from ordinary men in the level but not in the nature of his thinking, such a change will not only not harm his less gifted schoolmates, but will help them to achieve the best development their ability permits.

Thus the prescription for a school is this - treat all your students as if they were great men in a small way.

In another passage [7] Ostwald indicates places where he

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[7] Lebenslinien, p.134

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thought real education was taking place. Two German institutions had won the respect of the world - the university and the kindergarten. These worked on the basis of freedom, and - in his own words -

> In both of them successful working was shown by the joy with which both pupils and teachers took part in them. The well known joylessness of the secondary school (the gymnasium) is by itself a proof of its futility.

It is because he knows what real work is. Chemistry fascinates him and he works hard at chemistry because of his delight in it.

Now we come to my first basic position. Suppose you have had this experience of loving a subject and working happily at it, and you see children being taught this subject in such a way that they dislike it, and have to be forced to do it, and the longer they do it the more they hate it. Two thoughts will occur to you. First, this is a crime against the children. Second, that even as a way of teaching the subject, of getting children through examinations or winning scholarships or preparing for a profession, it is an extremely inefficient method. A child who wants to know something will learn more in a minute than a child who under compulsion works grudgingly for an hour.

I often notice in the streets little groups of children arguing intensely and usually very intelligently about some question that concerns them. Our problem is to get the same thing happening inside the school.

Very young children are prepared to be interested in almost anything. When a child dislikes a subject, something has gone wrong. We will consider some possible causes.

One of the biggest causes is the lockstep. There still seem to be schools and school systems which work on the assumption that an entire class will work through a prescribed textbook at the same rate, which inevitably means that the quicker students are bored and the slower ones are lost. I heard some time ago of a child who came to Canada from overseas. He could already read and write before he entered Gr.1. However he was in a class where the rest of the children had never held a pencil in their hands and they were working at pothooks. The teacher insisted that this boy also do pothooks. This killed his interest in writing and in school generally. How surely if this teacher had had an ounce of imagination she could have found something for him to do that would have used his skill in writing, even if it was only copying sentences from some book in the room. We have so many difficult problems, when a child does not seem interested in any lesson at all; surely we should welcome it when an easy problem occurs - when a student has an interest in a subject and wants to work ahead at it, we should encourage him to do so. Yet very often this is not done, and we may well turn our best students into our worst.

The plight of the slower student under the lock step is as bad or worse. Either he is told he has failed - simply because it takes him 15 months to learn what other students learn in 10 - or he is promoted and is dragged along, getting more and more confused with work he cannot understand as the years pass.

One of the fashionable words at present is "continuous progress" and I am glad it is fashionable because it is educationally sound; it compares very favourably with some of the slogans of the "new maths" epoch.

Now, of course, even when you want each student to do work adapted to what he already understands, you often have difficulty in finding what stage he has reached. Even an experienced teacher can fall into the trap of trying to build on a concept which the child does not have. I ran into a difficulty of this kind myself in a class of future teachers; the work had been completely meaningless to them, because they did not have an idea that I had assumed every university graduate would have.

This problem is particularly acute in a system where each student works at his own pace. When the children come into a new class, or go from elementary to secondary school, the teacher asks himself, "How am I supposed to know what

The answer, I believe, lies in each child having an individual record sheet, which would accompany him from class to class in elementary school, and from elementary school to secondary school. Since we wish to teach skills based upon understanding, this record should particularly be concerned with what ideas, what concepts the child has mastered.

Some sheets are available to illustrate what I mean. Some time ago I tried to make a list, showing how many ideas were involved in arithmetic and what these ideas were. The resulting list was embodied in these record sheets.

I once had the following experience in a good elementary school where a real effort was being made to teach arithmetic well. In one classroom a girl was having a great difficulty with questions of the type 137 multiplied by 4. I asked her to show me what the question meant. What did 137 mean? There were bundles of coffee sticks available. When I asked her to set out 137 coffee sticks, I discovered that she did not know that ten bundles of ten made a hundred. Naturally it was quite impossible for her to understand an explanation of the multiplication procedure.

You may remember from my quotation of Mr.Polly "he was always doubtful whether it was eight sevens or nine eights that was sixty three". The real sting is in the comment "he knew no method for settling the difficulty". You will agree that eight sevens is one of the most mysterious items in the multiplication table. If there is a result that a child does not know, this will probably be it. So let us look at the question; suppose you do not know what eight sevens are, what is involved in thinking it out.

I have here a very old, very simple device - a board with 100 nails or hooks, arranged in a square, and some rings to hang on it. I would begin by asking a child, "What do you understand by the question? Show me what eight sevens means." I would expect the child to put seven rings on each of eight rows, or in some other way to show what eight sevens represented. Next the child has to express this number in our usual standard form, as so many tens and so many ones. He finds he can fill five rows with rings, and have six in the next row, so the answer is 56. Thus the question can be answered by a simple experiment.

It is really an astonishing achievement of our educational system to have made such a simple thing into a mystery.

I do not wish to imply that every time a child has to deal with eight times seven he should need to get out this piece of apparatus. But I do think it provides a foundation for understanding the basic needs, and it can lead to ways of thinking the result out. For instance, by placing a string against the board, you can see that 8 sevens breaks up into 8 fives and 9 twos. If the child knows these results he can see that 8 sevens is 40 + 16, and so 56. If he repeatedly thinks out this result, in time he will come to remember it immediately, and it will be imbedded in a logical structure in his mind.

The aim of teaching is to present complicated things in such a way that they become simple. Some of the reformers a dozen years ago seemed expert in making things complicated. We have to bear in mind that in the past arithmetic was taught so badly that many adults are in much the same stage - where arithmetic is concerned - as unfortunate Mr.Polly. This applies to many parents and many teachers. It is not their fault; they were taught in a way that made it almost inevitable. If someone has been so mishandled that he is terrified of arithmetic.

I don't think it is going to help him if you go to him and say, "Addition is really quite simple, it is only the union of disjoint sets." In fact union of sets seems to be a singularly inappropriate way of approaching addition. We are still seeing it at the beginning of books for young children; I think the authors of these books are victims of a confidence trick played on them by a minority of college mathematicians. And in any case, young children should not learn the basic concepts from pictures in books; they should learn them from handling actual objects.

Most of the tests in my concept list require the child to demonstrate with actual objects the meaning of basic ideas such as addition, subtraction, multiplication, division, numeration, or fraction. A teacher working on the basis of this list would have to teach the child to associate such words with actual situations, so that pure rote learning would become almost impossible. This in itself would be a great advance.

It would not be necessary for the teachers to take a long course of re-training. Even a teacher who had grown up under a system of pure rote learning would come to realise, after using this system of recording a child's progress, that every question in arithmetic could be thought out.

I would like to hear of it if anyone can find anywhere a child or a teacher not able to do and to understand what I did on this board with eight sevens. I exclude of course cases where the board performs a useful diagnostic service by showing that the child does not know what multiplication is, or that 56 means 5 tens and 6 ones.

In connection with the use of concrete materials, a very significant thing was observed during the second world war, which seems to have attracted very little attention on this continent. It was found that boys, who had done very badly at mathematics at school, did very well when the mathematics arose in some actual situation. I suppose there were two reasons for this; when mathematics was needed in order to understand some machine, the boys could see what it was <u>about</u> and what it was <u>for</u>. The Scottish Education Department issued a report on this with some very plain speaking. It said, in effect, that most failures in mathematics were not due to the stupidity of the pupils but to the incompetence of the schools.

In the last fifteen years or so there has been a remarkable movement in the British elementary schools in line with such thinking. There is an attempt to involve the children in real life situations which interest the children and from which they can learn mathematics. For instance, they may go out into the playground and try to determine the speeds of passing cars.

I have noticed opposite ends of the same problem arising in Canada and in England. In Canada a teacher may say, "I would like to get my children to understand division better, but I cannot think of enough activities that will lead them to understand division." In England the idea is widely accepted that children should be moving around "exploring the environment" and not just sitting at desks, but the complaint is heard that some teachers do not understand the purpose of the activities; the children are having a good time but they are not learning mathematics. It is further objected that the activity approach is haphazard, that it fails to produce a systematic learning of mathematics.

Both problems amount to this; we want the activities to be organised around a systematic mathematics curriculum. Now this is something on which teachers definitely need help. I know from experience how hard it can be to devise such a programme. When I was at Leicester in the 1940's we had a group of very competent, and devoted teachers, who worked out experiments and activities for mechanist apprentices who were taking a prescribed course in mathematics. It took three years of hard and often painful research to collect satisfactory materials.

What I would like to see is this. We would start from an analysis of what we were trying to teach - something like the Concept List I have circulated. We would then try to place against each concept a list of activities that might help the forming of that concept. A variety of activities would have to be given, so that teachers could choose those most suited to the locality where they lived and the individual tastes of their students. In the course of making such an index we would no doubt find that certain concepts were very short of appropriate activities. This would direct our attention towards devising activities for them.

Incidentally, I would like to emphasise that the activity approach does not mean that every lesson must take place in the playground. All that is necessary is that there should be enough activity to make the mathematical work meaningful and purposive.

In competent hands the activity approach produces wonderful results.

The Training of Teachers

Here we come to the heart of the problem - how are we to get competent teachers?

We have only to picture what mathematics lessons would look like in an ideal school to see the difficulty of the problem. There would be great flexibility. Some academically oriented children would be reading on their own at topics several years ahead of the present curriculum. Others, more physically minded, would be learning mathematics through scientific experiments or practical activities of some kind. All the children would be thinking about mathematics and attacking problems, not just memorising textbooks, though of course they would master the routines of mathematics in order to solve their problems successfully.

The difficulty is clear - no teacher grew up in a school like this.

Where in fact are teachers trained? One might say - in teachers' colleges or colleges of education. But this is not so.................................. .... In Toronto most elementary school teachers go from secondary school to a single year in teachers' college. It is planned that they should have a year in university as well. My feelings on this are mixed. I want elementary school teachers to know a lot. But is there any worse model for teaching in elementary school than what happens in many university lecture rooms?

A teacher's picture of what education is comes from the classes in which he has been taught. Here the overwhelming importance of elementary education is seen. Not only are more years spent in this stage than in any other, but this happens at an early and impressionable age, when lifelong attitudes are being formed. I know a teacher in the United States who asked secondary school students whether they liked mathematics or not, and who had made them feel that way. Some liked it, some disliked it, but in all cases the feeling has been created by some teacher in elementary school; no secondary school teacher was mentioned by any pupil.

The key, then, to any national advance lies in the elementary school. At the Commonwealth Conference in Trinidad I put forward a suggestion; whether it is something that actually can happen I do not know. My proposal was that the minimum mathematical programme for secondary school should be that every pupil left with the basic equipment of a good elementary school teacher.

This meant the following objectives in the following order of importance. In our culture people are afraid of mathematics. This has to be changed before any real advance is possible. The first objective is therefore concerned with emotions - (1) the student should enjoy mathematics and not be afraid to think about mathematics. Then (2), he should be able to grasp mathematical results informally, pictorially or in terms of a concrete situation. (3) he should have worked with mathematics in connection with simple scientific laws and with the environment generally. (3) In this way he should have acquired as much knowledge as possible of arithmetic, algebra, geometry, and perhaps other mathematical subjects.

In some cases, to achieve this would call for very radical and unorthodox procedures.

One thing the diagram brings out is the immense interdependence of educational institutions. You sometimes hear university professors complaining about the limitations of secondary school teachers. They seldom seem aware how much of this is due to the very unsatisfactory teaching procedures in universities. In the same way, secondary school teachers sometimes sound off about elementary school teachers. But they had these teachers for four or five years as secondary school students - were they not able to influence them at all in this time?

Certainly any future advance must be based on co-operation between institutions at different levels. An elementary school teacher has to teach all subjects. A brilliant child, whose interest is concentrated in one subject, will reach a level in that subject far beyond that one can reasonably expect from an elementary school teacher. And yet this child will sometimes need help. Secondary schools should have an arrangement with elementary schools in their locality, by which such students can receive occasional help and advice on what to read. In the same way, universities should assist students in secondary school who wish to forge ahead. I know of instances in which this is happening.

At Central Technical School in Toronto, Mr.George Buchanan runs a small science and mathematics club for elementary school children who live within walking distance. The club is purely voluntary and the children come after school on Mondays. Last year several students at the College of Education volunteered to help with this club. It should be much easier for them to co-operate with elementary schools in the future, when they have had this experience of working with younger children. It is also a useful experience of working in an informal atmosphere.

Another group of students at the College of Education ran a club for gifted children at an elementary school. Perhaps the most enterprising students were those who volunteered to run a club for Restless Boys, who were selected for us by a school principal. The idea was to operate in the school playground more or less as an athletics club, but to keep elaborate records of achievements -

graphs, averages and so forth - and to see how much mathematics would grow naturally out of the boys' activities. An interesting thing happened here. Almost immediately the students reported that there were two kinds of restless boys in grade 5 - those who were physically active and wanted to run about (these were the ones we had in mind when we started the club) - and those who were restless because they were capable of work about four years ahead of what anybody was asking them to do. Evidently this school needed two clubs rather than one.

In such clubs young teachers can learn the technique of teaching children through their interests. Since the experiment takes place outside school hours, there is no danger of disturbing other classes; there is no discipline problem; there is no curriculum that has to be covered within a given time.

These clubs have two purposes; on the one hand, they give future teachers the experience of working with flexible arrangements; on the other hand, there is the hope that some of the children in these clubs may become teachers, and that their experience in the club may become part of their picture of what a lesson should be like.

I have a club of my own for secondary school students. As there are about 60 schools within reach of the university we have to ration each school to 4 students. This gives an audience between two and three hundred. The schools release students early and we meet at 3 o'clock in the centre of the city, once a month in the winter. Naturally the club contains very good mathematicians and lecturers rattle along at a rate that would not be possible otherwise. Here again my hope is that some of the club members will end up in teaching. I was very pleased the other day when a student at the College of Education approached me and said, "Students do so little calculus in secondary school. They could go much further into it. I am thinking of starting a club in my old school to do calculus." He added, "I was in your club four years ago." So it is possible to get the process moving and growing.

Of course a club for 250 students in a city of 2,000,000 is merely scratching the surface. We have been seeking ways of spreading the influence, perhaps through written or printed materials. Perhaps the gifted students will be the first to respond to these, but I would hope to produce some sort of publication that would interest students of all kinds - academic or technical, elementary or secondary.

One thing emerges clearly - if we are to achieve a flexible system, students must be taught, from Grade 1 on, to read and study for themselves. The universities use the lecture system, which originated before printing was invented and is now thoroughly obsolete. It would be far better to give the students a book to read, and use the class periods to answer questions and discuss difficulties. The reason professors do not do this is that <u>they do not believe the schools have taught the students to read</u>.

The argument in brief is this - the lockstep system produces the most awful casualties; if we are to have flexibility, the students must become accustomed to working independently; they will only do this if they are interested in the work.

As I said earlier, I do not want to see any sudden or dramatic change. When a new direction is taken, mistakes are bound to be made; it is best to make them on a small scale. What is heartbreaking is when a young teacher, who has a vision of what might be done, is trapped in a school where nothing can be done.

I would be happy if we had some arrangement by which young teachers, who wanted to move in the direction I have indicated, went to school principals who would give them some scope to try out new ideas, and would so organise things that inevitable mistakes would not become, or be regarded as, disasters. For instances, in most schools there are some students who seem to have no hope at all of succeeding under the present arrangements. They can hardly be worse off with an unconventional approach; on the other hand, it could be their salvation. At the other end, there are brilliant scholars whose whole desire is freedom to forge ahead on their own. We are only taking a very small risk if we give them the chance to do so.

A principal once asked me, "Do we need to take all this trouble to teach mathematics? Do people really need mathematics in life?" This is a very sound question.

I will give two answers to this question. The first is a very defensive, backs-to-wall kind of answer; the second is nearer to the actual facts.

First then, I do not believe that, in the climate of this age, any school system is going to say, "We are not going to teach mathematics at all." If then you are going to teach it, you will teach it well or badly. If you teach it badly, you will be injuring the minds and souls of your students. So it would be desirable to teach it well.

The second, more positive answer - we are living in an increasingly complex civilization. The beginnings of nearly every science demand understanding of arithmetic and some elementary algebra. This is true whether you wish to study statistics or chemistry, electricity or heredity. With automation steadily wiping out routine work, it may be that an understanding of such things will be necessary even to find employment. It will certainly be necessary to understand the issues that confront every citizen. Atomic physics and biochemistry have invaded our everyday life. Do television sets produce a dangerous quantity of X-rays? Should pregnant women take aspirin? Are the great lakes being killed because detergents include phosphates? Should fluorides be put in the drinking water? To-day the average man has to judge between the conflicting views of experts on such questions and cast his vote. Today you cannot be a humanist unless you are in some measure a scientist.