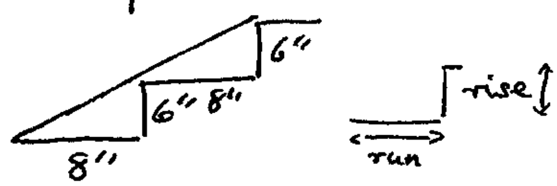
Trigonometry is about sizes, shapes and positions. For instance, if a staircase has steps with a rise of 6 inches and a run of 8 inches, and a plank rests on it, as shown, the question may arise – at what angle is this plank?



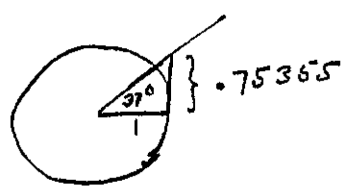
To answer this question, we have to use a table of tangents. This will contain entries such as the following.

Angle	Tangent
36°	.72654
37°	.75355
38°	.78129

For our stairs, rise divided by run $= 6 \div 8 = 0.75$. In the table we see .75355 which is nearly the same as 0.75. This tells us that the plank would make an angle of, very nearly, 37°.

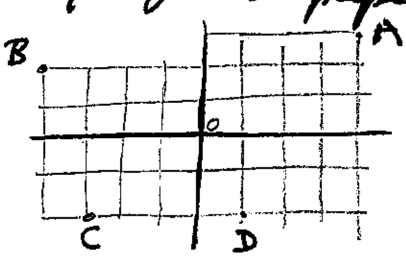
A table of tangents, then, tells us what number, rise divided by run, corresponds to any angle. You may have all sorts of questions about how such tables are obtained and how they work. The construction of the tables calls for harder theory than using them does. For the present, it is wise to take the tables on faith, and concentrate on seeing how they are used.

The origin of the name 'tangent' is simple to understand. If you have a circle of radius 1, and an angle of 37° at the centre, the length of the tangent will be .75355, so this number is called 'the tangent of 37°'. For short, we write $.75355 = \tan 37°$.



The table of tangents is one of the three tables commonly used in trigonometry. The other two are sine and cosine.

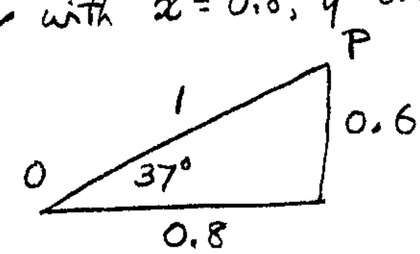
Sine and cosine arise very naturally in describing position. One way of describing position, of course, is by squared paper, with x for distance across and y for distance up. There is the usual agreement that x is + when we go to the right, − if we go left, while for y, up is + and down is −. So, in this diagram A has $x = 4, y = 3$; B has $x = -4, y = 2$: C has $x = -3, y = -2$ and D has $x = 1, y = -2$.



If this was a map, and a sailor wanted to sail from O to A, he would not want to go 4 miles east and then 3 miles north; he would want to know how far it is from O to A, and at what angle. Really, I think this is a good way to look at the central idea of trigonometry — how do you interpret squared paper descriptions in terms of distances and directions, and vice versa?

A natural thing to study is — if you go 1 mile at any specified angle, how much is this to the East and North? (Using minus numbers, if your angle takes you West or South.)

For instance if we go 1 mile at the angle we met earlier — it is really 36.8699°, but I will call it 37° — we find this takes us 0.8 mile East and 0.6 mile North. So, if we start at the origin, O, and go unit distance at the angle 37°, we reach the point with $x = 0.8$, $y = 0.6$.

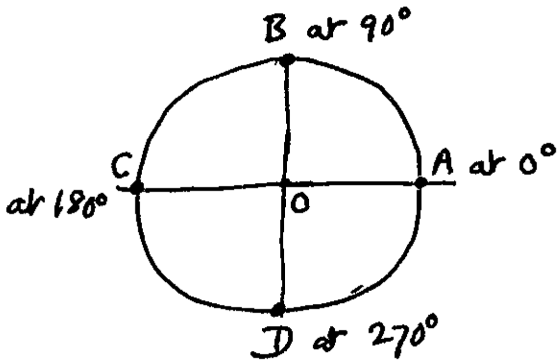


Such results are needed very often, and so to save having to explain all this each time, names have been invented. The number x that comes is called the <u>cosine</u> of the angle, the number y is called the <u>sine</u> of the angle — usually abbreviated to <u>cos</u> and <u>sin</u> in writing.

So $\quad 0.8 = \cos 37°. \quad 0.6 = \sin 37°.$

All we have here are two new names for quite simple ideas.

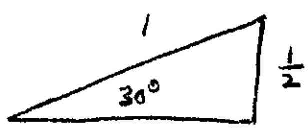
The point P of course lies on the circle with centre O and radius 1.



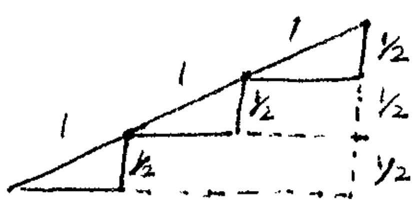
Four particularly simple points on this circle are A at 0°, B at 90°, C at 180° and D at 270°.
We can give the cosines and sines of these angles simply by reading off the values of x and y at A, B, C and D.

A is at $0°$ and has	$x=1,\ y=0$ so	$\cos 0° = 1$,	$\sin 0° = 0$.
B . . . $90°$	$x=0,\ y=1$	$\cos 90° = 0$	$\sin 90° = 1$.
C $180°$	$x=-1,\ y=0$	$\cos 180° = -1$	$\sin 180° = 0$.
D $270°$	$x=0,\ y=-1$	$\cos 270° = 0$	$\sin 270° = -1$.

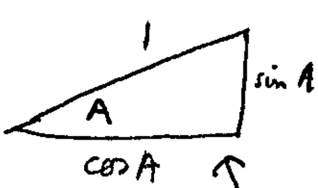
There is a very simple result for $\sin 30°$. $\sin 30° = \frac{1}{2}$

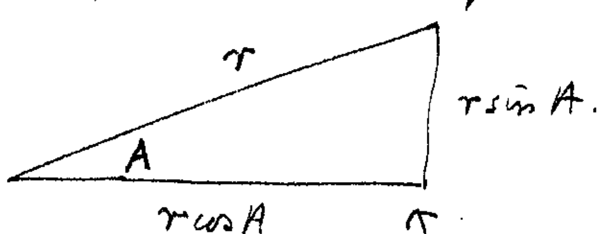


Knowing the result for a line of length 1 allows us to work out the result for a line of any length. If we draw 3 steps of a staircase having angle 30° we get this:-



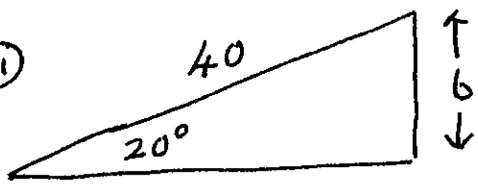
so if you travel a distance 3 at angle 30° you rise $3 \times \frac{1}{2}$. If you go a distance r, you rise $r \times \frac{1}{2}$.



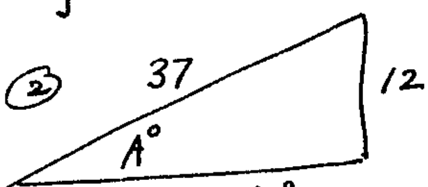


If we enlarge this picture r times we get this one, which should be thoroughly memorized.

You can work yourself into it by doing a number of little questions like the following :–

(1) 

Find b.

(2) 

What is A° ?

(1) If you compare this with $*$ you see
$r = 40 \quad b = r \sin 20°$, so $b = 40 \sin 20°$.
The table gives $\sin 20°$ as $.3420$, so $b = 40 \times .34202$
$= 13.6808$.

(2) Compare with $*$. $r = 37$

$$r \sin A° = 12$$

$$\therefore \quad 37 \sin A° = 12$$

$$\therefore \quad \sin A° = \frac{12}{37} = .32432$$

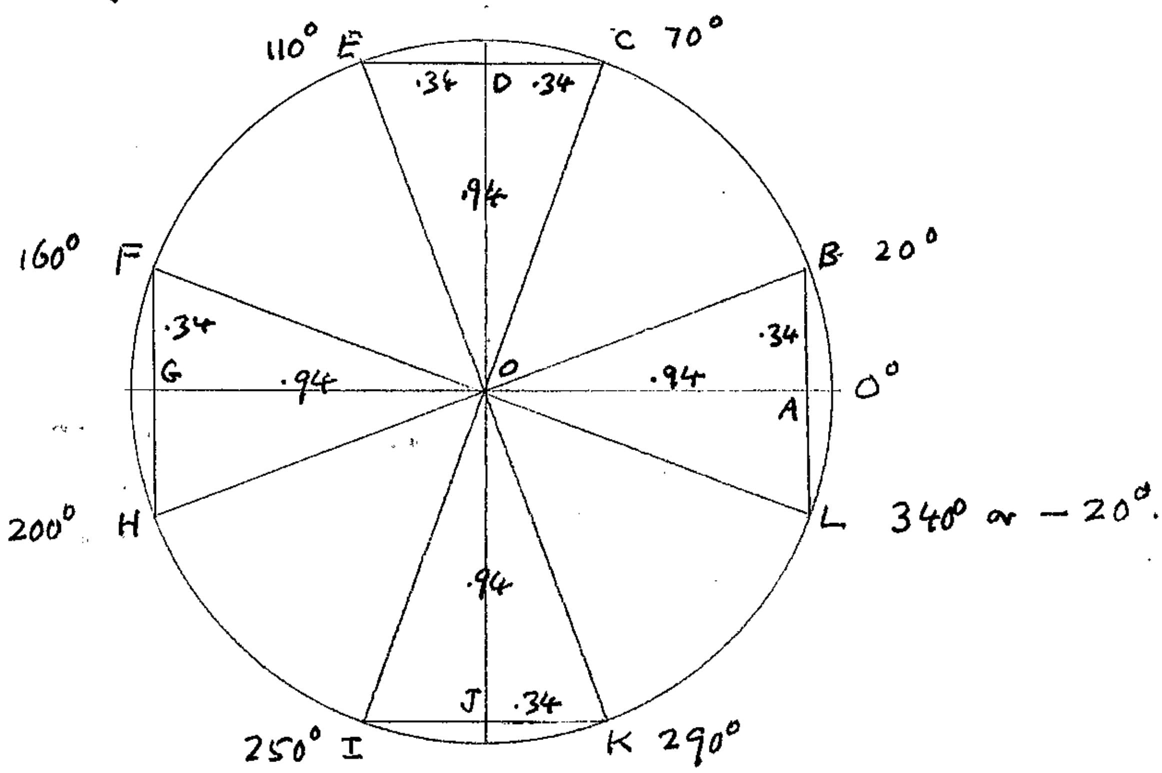
From the tables $\sin 19° = .32557$
$\sin 20° = .34202$
$\sin 18° = .30902$

so $A°$ is just a little less than $19°$
(It can be found more accurately, but for the moment this is good enough)

You should find a supply of simple questions of this kind in any secondary school textbook. They do not offer much variety at this stage — they just help to work into your mind the meaning and use of sin and cos.

By the way, a scientific calculator, which is not unduly expensive, can replace tables — it has keys marked sin, cos, tan.

On page 3, I considered $\sin 180^\circ$ and $\cos 180^\circ$ etc. Tables usually go only from 0° to 90°, but in some work you need to know, perhaps, $\sin 160^\circ$. This can be found from the tables for 0° to 90°, with the help of a diagram I call the Maltese Cross.



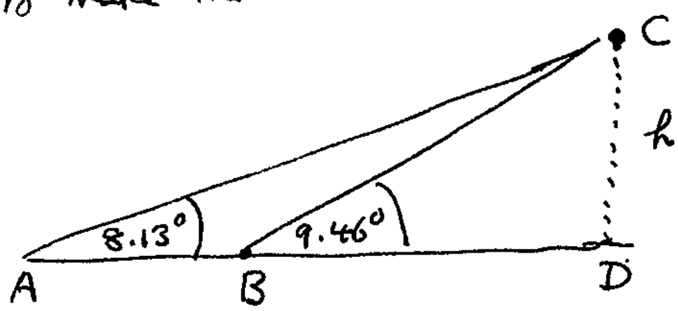
In this picture I have taken B at 20°, — the argument would work equally well for other angles. The other corners, C, E, F, H etc are at $90^\circ - 20^\circ$, $90^\circ + 20^\circ$, $180^\circ - 20^\circ$, $180^\circ + 20^\circ$ etc. Obviously the triangles OAB, ODC, ODE etc all have the same measurements. If we know the co-ordinates of B, we can easily write down x and y for C, E, F, H etc and these give us the sines and cosines of the angles they are at. To two places of decimals, $\sin 20^\circ = .34$ and $\cos 20^\circ = .94$. (I am only using two places, so as not to have crowded figures on the diagram.) We can get from O to C by going .94 north, then .34 east, so for C $x = .34$, $y = .94$. (There is now law that says you must go east first and then north! You can check on squared paper that you get the same co-ordinates whichever you do.) For E, $x = -.34$, $y = .94$. Similarly, you can write down the coordinates of F, H .....-

You can tabulate the results.

	20°	70°	110°	160°	200°	250°	290°	340°
x: cosine	.94	.34	−.34	−.94	−.94	−.34	.34	.94
y: sine	.34	.94	.94	.34	−.34	−.94	−.94	−.34

You may notice that $\cos 70^\circ = \sin 20^\circ$ and $\cos 20^\circ = \sin 70^\circ$. Quite generally $\cos A^\circ = \sin (90-A)^\circ$, so books of tables often give sines, but do not bother with cosines. If you want, say, $\cos 35^\circ$, you are expected to find $90^\circ - 35^\circ = 55^\circ$ and look up $\sin 55^\circ$.

Trigonometry seems to be only about triangles containing a right angle, but by fitting right-angled triangles together all kinds of questions can be dealt with. Here's an example which I've faked to make the arithmetic almost non-existent.



A and B are 1 mile apart. An observer at A notices a balloon, C, at an angle of elevation of 8.13°. For an observer at B, the angle of elevation is 9.46°. How high is the balloon? I chose 8.13° because $\tan 8.13^\circ = \frac{1}{7}$: this means $\frac{h}{AD} = \frac{1}{7}$ or $AD = 7h$.

Also $\tan 9.46^\circ = \frac{1}{6}$ so $\frac{h}{BD} = \frac{1}{6}$ so $BD = 6h$.

(Both of these are 'rise-over-run' for tangent of an angle.)

As $AD = 7h$ and $BD = 6h$, $AB = 7h - 6h = h$.

So the height of C is the same as the distance, AB, that is, 1 mile. The same method works equally well with numbers that could happen in practice.

You may find it worth while to read Chapter 13 of Mathematician's Delight, or at any rate the earlier part of it. Also look at school text books. A good way to learn any subject is to keep reading it in different books, choosing ones that appeal to you. Don't worry about understanding everything: let it gradually soak in.