Trigonometry is about sizes, shapes and positions. For instance, if a staircase has steps with a rise of 6 inches and a run of 8 inches, and a plank rests on it as shown, the question may arise: or what angle is this plank?

To answer this question, we have to use a table of tangents. This will contain entries such as the following:

<table>
<thead>
<tr>
<th>Angle</th>
<th>Tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>36°</td>
<td>0.72654</td>
</tr>
<tr>
<td>37°</td>
<td>0.75355</td>
</tr>
<tr>
<td>38°</td>
<td>0.78129</td>
</tr>
</tbody>
</table>

For our stairs, rise divided by run = 6 ÷ 8 = 0.75. In the table we see 0.75355 which is nearly the same as 0.75. This tells us that the planks would make an angle of, very nearly, 37°.

A table of tangents then tells us what number, rise divided by run, corresponds to any angle. You may have all sorts of questions about how such tables are obtained and how they work. The construction of the tables calls for harder theory than using them does. For the present, it is wise to take the tables on faith, and concentrate on seeing how they are used.

The origin of the name 'tangent' is simple to understand. If you have a circle of radius 1, and an angle of 37° at the centre, the length of the tangent will be 0.75355, so this number is called 'the tangent of 37°'.

For short, we write \( \tan 37° = 0.75355 \).

The table of tangents is one of the three tables commonly used in trigonometry. The other two are sine and cosine.
Sine and cosine arise very naturally in describing position.
One way of describing position, of course, is by squared paper,
with \( x \) for distance across and \( y \) for distance up. There is the usual agreement that \( x \) is + when we go to the right, \( y \) is 0 when we go left;
while for \( y \), up is + and down is .
So, in this diagram A has \( x = 4, y = 3 \);
B has \( x = -4, y = 2 \); C has \( x = -3, y = -2 \);
and D has \( x = 1, y = -2 \).
If this was a map, and a sailor wanted to sail from
0 to A, he would not want to go 4 miles east and then
3 miles north; he would want to know how far it is from 0
to A, and at what angle. Really, I think this is a
to A, and at what angle. Really, I think this is a
natural thing to study is — if you go 1 mile at any
specified angle, how much is this to the East and North?
(Using minus numbers, of course; you West or South.)
For instance, if we go 1 mile at the angle we met
earlier — it is really 36.869°, but we will call it 37° —
we find this takes us 0.8 mile East and 0.6 mile North.
So, if we start at the origin, 0, and go unit distance
at the angle 37°, we reach the point with \( x = 0.8, y = 0.6 \).
Such results are needed very often,
and so to save having to explain all
this each time, names have been
invented. The number \( x \) that
comes is called the cosine of the angle;
the number \( y \) is called the sine of the angle — usually
abbreviated to \( \cos \) and \( \sin \) in writing.
So \( 0.8 = \cos 37° \), \( 0.6 = \sin 37° \).
All we have here are two new names for quite
simple ideas:
The point P, of course, lies on the circle with
centre 0 and radius 1.
Four particularly simple points on this circle are A at 0°, B at 90°, C at 180° and D at 270°.
We can give the cosines and sines of these angles simply by reading off the values of x and y at A, B, C and D.

A at 0° and has x = 1, y = 0 so \( \cos 0° = 1 \), \( \sin 0° = 0 \).
B at 90° x = 0, y = 1 \( \cos 90° = 0 \), \( \sin 90° = 1 \).
C at 180° x = -1, y = 0 \( \cos 180° = -1 \), \( \sin 180° = 0 \).
D at 270° x = 0, y = -1 \( \cos 270° = 0 \), \( \sin 270° = -1 \).

There is a very simple result for \( \sin 30° \): \( \sin 30° = \frac{1}{2} \)

Knowing the result for a line of length 1 allows us to work out the result for a line of any length. If we draw 3 steps of a staircase having angle 30° we get this:

So if you travel a distance 3 at angle 30° you rise \( 3 \times \frac{1}{2} \). If you go a distance \( r \), you rise \( r \times \frac{1}{2} \).

If we enlarge this picture we get the one, which should be thoroughly memorized.
You can work yourself into it by doing a number of little questions like the following:

1. \[ \begin{array}{c} \text{Find } b. \\ \hline \text{40} \\ \text{20°} \end{array} \]

2. \[ \begin{array}{c} \text{What is } A°? \\ \hline \text{37} \\ \\ \text{12} \end{array} \]

1. If you compare this with \( \gamma = 40 \), \( b = r \sin \, 20° \), so \( b = 40 \times 0.3420 = 13.6808 \).

2. Compare with \( \gamma = 37 \), \( r \sin A° = 12 \).

\[ \frac{37 \sin A°}{37} = \frac{12}{37} \approx 0.3243 \]

From the tables, \( \sin 19° = 0.32557 \), \( \sin 20° = 0.34202 \), \( \sin 18° = 0.30902 \).

so \( A° \) is just a little less than 19°.

(If it can be found more accurately, but for the moment this is good enough.)

You should find a supply of simple questions of this kind in any secondary school textbook. They do not offer much variety at this stage – they just help to work into your mind the meaning and use of \( \sin \) and \( \cos \).

By the way, a scientific calculator, which is now remarkably expensive, can replace tables – it has keys marked \( \sin \), \( \cos \), \( \tan \).
On page 3, I considered \( \sin 180^\circ \) and \( \cos 180^\circ \), etc. Tables usually go only from \( 0^\circ \) to \( 90^\circ \), but in some work you need to know, perhaps, \( \sin 160^\circ \). This can be found from the tables for \( 0^\circ \) to \( 90^\circ \), with the help of a diagram I call the Maltese Cross.

In this picture I have taken \( B \) at \( 20^\circ \); the argument would work equally well for other angles. The other corners, \( C, E, F, H \) etc. are at \( 90^\circ - 20^\circ, 90^\circ + 20^\circ, 180^\circ - 20^\circ, 180^\circ + 20^\circ \) etc. Obviously the triangles \( OAB, ODC, OPE \) etc. all have the same measurements. If we knew the coordinates of \( B \), we can easily write down \( x \) and \( y \) for \( C, E, F, H \) etc. and these give us the sines and cosines of the angles they are at. To find place of decimals, \( \sin 20^\circ = .34 \) and \( \cos 20^\circ = .94 \). (I am only using two places, so as not to have crowded figures on the diagram.) We can get from \( O \) to \( C \) by going \( .94 \) north, then \( .34 \) east; so for \( C \) \( x = .34 \), \( y = .94 \). (There is now law that says you must go each time in the same direction as the coordinates whatever you do.) For \( E \), \( x = -.34 \), \( y = .94 \). Similarly, you can write down the coordinates of \( F, H \). . . .
You can tabulate the results:

\[
\begin{array}{cccccccccccc}
20^\circ & 70^\circ & 110^\circ & 160^\circ & 200^\circ & 250^\circ & 290^\circ & 340^\circ \\
\sin x & .94 & .34 & -.34 & -.94 & -.94 & .34 & .94 \\
\cos y & .34 & .94 & .94 & .34 & -.34 & -.94 & -.94 \\
\end{array}
\]

You may notice that \( \cos 70^\circ = \sin 20^\circ \) and \( \cos 20^\circ = \sin 70^\circ \).

Quite generally, \( \cos A^\circ = \sin (90^\circ - A)^\circ \), so both of table entries give sines, or rather cosines with sines.

Often given sines, we do not bother with cosines.

If you want, say, \( \cos 35^\circ \), you are expected to find \( 90^\circ - 35^\circ = 55^\circ \) and look up \( \sin 55^\circ \).

Trigonometry seems to be only about triangles containing a right angle, but by fitting right-angled triangles together all kinds of questions can be dealt with. Here's an example which I've failed to make the arithmetic almost non-existent.

A and B are 1 mile apart. An observer at A notices a balloon, C, at an angle of elevation of \( 8.13^\circ \), for an observer at B, the angle of elevation is \( 9.46^\circ \). How high is the balloon? I chose 8.13° because \( \tan 8.13^\circ = \frac{1}{7} \); this means \( \frac{h}{AD} = \frac{1}{7} \) or \( AD = 7h \).

Also \( \tan 9.46^\circ = \frac{1}{6} \), so \( \frac{h}{BD} = \frac{1}{6} \) so \( BD = 6h \).

(Both of these are 'rise-over-run' for tangents of an angle.)

So the height of C is the same as the distance AB, that is, 1 mile. The same method works equally well with numbers that could happen in practice.

You may find it worth while to read Chapter 13 of "Mathematician's Delight," or any other the earlier part of it. Also look at school text-books. A good way to learn any subject is to keep reading it in different books, choosing ones that appeal to you. Read many about understanding everything, let it gradually sink in.