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THE MATHEMATICIAN AS A PERSON .

The title for this talk was given to me, as something you would like me to talk about. I do not know if you hoped I would give you a portrait of a mathematician, what he looks like, what his habits are, and so forth. If so, you will be disappointed. In Moslem theology, you will find the following characterization of Allah, of God ; whatever you think God is, he is not that. A similar statement could be made about mathematicians, and in a double sense ; first, mathematicians have been of such varied types, that any general statement about them is likely to be untrue of some mathematicians ; second, the public picture of a mathematician is so distorted that it is largely untrue for all mathematicians.

First of all the nature of mathematical ability : you will often hear people say that mathematical ability is a very special gift, which they do not have. This is entirely contrary to the evidence. Mathematics is largely a "high g" subject - that is to say, a subject requiring general intelligence. An Australian enquiry defined mathematical intelligence as the ability to hold a large number of facts in mind, and to see the relations between them. But this surely is what you need to learn chemistry or to write a history of the world.

The reason why mathematics is thought of as a special gift is bad teaching. In many schools something is taught in mathematics lessons which is not mathematics at all, but is an entirely different and much harder subject. Failure to cope with this artificial monstrosity leaves students with a sense of defeat, and this creates an emotional block, a fear of the subject, which creates an effect of stupidity at mathematics. Indeed, the more intelligent and the more sensitive a person, the more liable he is to this kind of emotional disturbance. This is why one meets, for example, brilliant classical scholars who claim to be mathematical morons.

Since mathematics calls for general intelligence, it is by no means certain that someone with mathematical potentiality will in fact become a mathematician. Gauss, one of the three or four greatest mathematicians that have ever lived, hesitated whether to become a mathematician or a philologist. Hamilton by the age of five could read Latin, Greek and Hebrew, and had added Persian, Arabic, Chinese and various Indian languages by the time he was thirteen. In his teens he worked at the classics, which were necessary for entrance to university in 1820, and only read mathematics for amusement. The childhood of Fourier is remarkable. He became an orphan at eight, and came under the protection of a bishop. I quote Bell - (Men of Mathematics ,chapter 12)

"By the age of twelve he was writing magnificent sermons for the leading church dignitaries of Paris to palm off as their own. At thirteen he was a problem child, wayward, petulant and full of the devil generally. Then, at his first encounter with mathematics, he changed as if by magic. "

Many mathematicians have shown marked linguistic or literary ability. In American colleges there is a widespread belief that unless a student does well in verbal aptitude tests he is unlikely to do well in mathematics. But this, I think, reflects rather the wordiness of American professors than the nature of mathematical thinking. I will balance my examples of literary and linguistic mathematicians by mentioning Hadamard, a mathematician of the first rank, he said that he never thought in words, and had the utmost difficulty in finding words to communicate his discoveries to others. Mathematical thinking can be completely non-verbal.

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Pale academics.

A widely held belief, concerning not only mathematicians but scholars of all kinds, is that men take up scholarly work because they are of weak physique. This belief is almost entirely incorrect. Galton (I am quoting from memory) said that if you collected together men of admitted genius you would find that you had a collection of superb animals. This view was put forcibly by Sylvester in an address to the British Association - I quote, compressing in places -

> Leibnitz lived to the age of 70 ; Euler to 76 ; Lagrange to 77 ; Laplace to 78 ; Gauss to 78 ; Plato ... lived to 82 ; Newton, the crown and glory of his race, to 85; Archimedes was 75 (Sylvester points out that he was killed by a soldier) ; Pythagoras.... (if we may credit the tradition) after spending 22 years studying in Egypt and 12 in Babylon, opened school when 56 or 57 years old, married a young wife when past 60, and died, carrying on his work with energy unspent to the last, at the age of 99. The mathematician lives long and dies young ; the wings of his soul do not early drop off, nor do its pores become clogged with the earthy particles blown from the dusty highways of vulgar life.

Sylvester himself lived to the age of 83. In passing, I would comment on his phrase that the mathematician "lives long and dies young". This I suppose reflects the well known Greek saying that those whom the Gods love die young, and brings out its real meaning - those whom the Gods love reamin youthful right until the time when they die.

Sylvester quotes a list of mathematicians who lived to great ages. To be convincing, one ought to show that one could not quote an equally long list of mathematicians who died early. In fact, to get a proper statistical argument, one ought to collect a comprehensive list of mathematicians and tabulate the ages at which they died. This I have not done - some of you may wish to fill this gap for yourselves. I can think of some mathematicians whose lives were short, but special circumstances account for many of these. Galois died at the age of 21, but he was killed in a duel. Abel died at 27, but poverty was an important factor. He was the son of a clergyman in Norway ; his father died when he was 18, and he had to support his mother and six other children as well as himself, in a poor country . Riemann died at the age of 40 ; he also was the son of a clergyman. He went to university in Germany at a time when it was assumed that only the moneyed classes were interested in learning, and junior university teaching was regarded as an honour and was unpaid. When Riemann was 29, a special concession was made to him ; he was paid one-tenth of a professor's salary in recognition of his unusual merit. Not until the age of 33, when a professorship became vacant, did he earn a living wage.

There are certainly many examples of mathematicians who showed great powers of physical endurance. Weierstrass for many years was an unknown schoolteacher, busy with the school's work during the day, a pleasant social companion at drinking parties during the evenings, and staying up at night to read mathematics and do original research. Of Monge Bell writes, telling how he held at the same time the professorship of physics and that of mathematics - "The double work did not bother him at all. Powerfully built and as strong of body as he was of mind, Monge was always capable of doing three or four men's work and frequently did".

Another mathematician with marked powers of endurance was Poncelet, who made very original contributions to geometry, in a Russian prison. He was a young officer in Napoleon's army at the time of the famous retreat from Moscow. Bell describes his experiences as follows -

"Among those left for dead on the frozen battlefield was young Poncelet. His uniform as an officer of engineers saved his life. A searching party, discovering that he still breathed, took him before the Russian staff for questioning.
"As a prisoner of war the young officer was forced to march for nearly five months across the frozen plains in the tatters of his uniform, subsisting on a meagre ration of black bread. In a cold so intense that the mercury of the thermometer frequently froze, many of Poncelet's companions in misery died in their tracks, but his more rugged strength pulled him through, and in March 1813 he entered his prison at Saratov on the banks of the Volga."

I do not know if you are inclined to be sceptical about the mercury freezing in the thermometer. This would happen at about 40 degrees below zero Fahrenheit, which is a perfectly possible temperature,

Mathematicians and National Affairs.

Monge and Poncelet both lived amid the upheavals of the French Revolution and the Napoleonic Wars. What then is the attitude of a mathematician to the political and social disturbances that go on around him ? It has been maintained that he is completely insulated from them. Thomas Hill, in Volume 85 of The North American Review , wrote as follows -

"The pursuit of mathematical science makes its votary appear singularly indifferent to the ordinary interests and cares of men. Seeking eternal truths, and finding his pleasures in the realities of form and number, he has little interest in the disputes and contentions of the passing hour. His views on social and political questions partake of the grandeur of his favourite contemplations, and , while careful to throw his mite of influence on the side of right and truth, he is content to abide the workings of those general laws by which he doubts not that the fluctuations of human history are as unerringly guided as are the perturbations of the planetary hosts."

This gives a very clear picture of the mathematician above the battle. The only objection is that it is not true. Some mathematicians, in some epochs, may have felt like that. But one cannot generalize. Monge, for example, played a very energetic and courageous part in the affairs of his time. Before the Revolution, when the aristocrats were still in control. Monge had the responsibility for examining young men who wished to become officers in the French navy. In spite of protests, he consistently turned down the incompetent sons of aristocrats, and During the revolutionary wars he organized the production of explosives and cannons. He ran of course the risk of being guillotined, with the rapid changes of political climate, and was in fact denounced and had to fly during the Terror.

Lazare Carnot is perhaps an even more striking example of a mathematician involved in the French Revolution. He was known as "the organizer of victory" ; he raised fourteen armies for the defence of France; earlier, at the battle of Wattignies, he took temporary command and won the battle. He had the usual experiencew of taking refuge abroad and coming back. He helped Napoleon for a time, but had the courage to resign when he realized Napoleon was trying to conquer an Empire. He did not approve of empires. Later, when France was again on the defensive, he once more offered his services. He wrote on mathematics whenever he was not doing something else. His son, Sadi Carnot, founded the science of mathematical thermodynamics. This also had some political motive. Many years ago I read Sadi Carnot's introduction to his work, and my recollection is that he said the strength of Britain rested on their development of the steam engine ; French scientists, by studying the basic principles of such machines, would be able to increase the strength of France.

Galois is well known as a mathematician of intense and radical political convictions. The duel, in which he died, seems to have been a consequence of these.

Monge,Carnot and Galois were believers in progress, in change. I do not want to give the impression that all mathematicians were on this side. There have been plenty of mathematicians in the spectrum from conservative to reactionary. Some were merely conservative in opinion, and did little about it, like Gauss. Others were active and outspoken, like Cauchy. Cauchy was a child during the Terror French Revolution - he was born in 1789 - and grew up near starvation. He survived this and lived to be 68, another example perhaps of the tendency mathematicians have to survive. It is not surprising that this childhood left him with strong and reactionary views. Bell describes Cauchy as "worshipping the Bourbons and believing the dynasty to be the direct representatives of Heaven sent to govern France." The Bourbons fell in 1830. Cauchy, at the age of 40, threw up all his positions and went into voluntary exile with them. Later he returned to France, and was in periodic trouble with the government ; the mathematicians kept electing him to posts for which an oath of allegiance was required, and which Cauchy always refused to give. Cauchy won ; both under Louis Philippe and under Napoleon III he was allowed to hold offices without taking any oath.

One might be tempted to conclude that, on whichever side they are, mathematicians are men of consistency and political principle. But this will not work either. There is the famous case of Laplace, who survived all the changes of government in the French Revolution by the simple method of praising whichever government happened to be in power at the moment, and dedicating his latest mathematical work to it. One might conclude that Laplace was entirely without without principle. This would not be correct. There is a very well-known story about Laplace and Napoleon. Laplace had sent Napoleon a copy of the Mecanique celeste, in which Laplace

gave a mathematical theory of the solar system. Napoleon enjoyed putting embarrassing questions and he said to Laplace, "I understand you have written a large book on the universe and have never mentioned its Creator". In the words of Rouse Ball

"Laplace, who, though the most supple of politicians, was as stiff as a martyr on every point of his philosophy, drew himself up and answered bluntly" I did not have need of that hypothesis"

Laplace evidently was willing to offend people for the sake of his atheism, in which he really believed. In politics he presumably felt that France would be equally badly governed, whoever was in charge, and that it was not worth while getting into trouble through politics. Incidentally, at one time Laplace persuaded Napoleon to let him become Minister of the Interior. Laplace made a very bad job of it, and Napoleon had to sack him after six weeks. Not all mathematicians are organizers of genius.

Mathematicians and Religion.

Mention of Laplace's atheism suggests the question of whether mathematicians agree about religion. The answer, of course, is that they are as varied in this as in any other matter. Cauchy, whose politics have already been mentioned, was a bigoted and intolerant Catholic ; Weierstrass was a tolerant Catholic, Hermite began as a tolerant agnostic and ended as a tolerant Catholic. Newton was a devout Protestant, and, besides his scinetific works, wrote a long interpretation of prophecies in the book of Daniel and the Apocalypse. G.H.Hardy, the outstanding British mathematician of the early twentieth century, was a militant atheist. On one occasion he refused to enter a college chapel even to take part in a college election. Hardy, incidentally, was said to have three ambitions or dreams - to make a century in cricket at Lord s cricket ground, to prove Fermat's Last Theorem, and to lead a successful revolution. He did not realize any of these dreams. Mathematicians in other ages have been Jewish, Moslem or Greek and have shared in the religion or irreligion of their society. Ramanujan was of Hindu origin. Hinduism seems to be a very tolerant faith, in which, provided you observe certain ceremonies, you can believe just about what you like ; much like what in England is called Broad Church. Certainly any religious outlook that would try to unite all mathematicians in a single creed would have to be very broad church indeed.

Mathematicians and Mathematics.

We have thought about mathematicians in relation to various topics outside mathematics. What about mathematicians in their own profession ? What do they think they are doing ? How do they see their own activity ?

It is very noticeable that mathematicians, in talking about mathematics, nearly always compare it to poetry or some other form of artistic creation. In Moritz' anthology "On Mathematics and Mathematicians" there are 36 different quotations collected under the heading of "Mathematics as a Fine Art" ; most of these compare poetry, music and mathematics. Hardy's well known book "A Mathematician's Apology" opens with the words -

In [illegible]mard's very interesting book, "The Psychology of Invention in the Mathematical Field", he speaks of the way in which a mathematical argument is built up. He makes interesting comparisons. It resembles the mental process by which we recognize the face of a person. He compares his thinking to that of a chess player who plays several games at once, and can remember the various games because each game has , so to speak, its own personality, its own face. He continues

"Now, such a phenomenon necessarily occurs in invention of any kind. We saw it mentioned in Mozart's letter ; similar statements are issued by artists like Ingres or Rodin. Only, while the happily gifted Mozart does not seem to have needed any effort in order to see the unity of his work, Rodin writes, "Till the end of his task, it is necessary for the sculptor to maintain energetically, in the full light of his consciousness, his global idea, so as to reconduct unceasingly to it and closely connect with it the smallest details of his work. And this cannot be done without a very severe strain of thought." Similarly, any mathematical argument, however complicated, must appear to me as s single thing. I do not feel that I have understood it as long as I do not succeed in grasping it in one comprehensive idea, and, unhappily, as with Rodin, this often requires a more or less painful exertion of thought."

This comparison of mathematical and artistic creation xx runs right through the writings and remarks of mathematicians. There is a well known saying of Weierstrass that no mathematician can be a complete mathematician unless he is something of a poet. This idea is put even more pointedly in a German anecdote . Two men are talking. One says, "What happened to Schmidt ? He was planning to become a mathematician, wasn't he ?". The other answers, "He gave it up. He found he hadn't enough imagination." "What did he become then ?" "A poet."

Perhaps because they find so much poetry in their own subject, some mathematicians have little interest in actual poems. Landau used to say poetry was just as much nonsense whether you read it forward or backwards, and to prove his point he used to recite "Die Lorelei" backwards.

Another story about mathematicians and poetry concerns Babbage, who already in 1812 was working on the idea of automatic computers. I quote from The World of Mathematics, volume 3.

When Tennyson wrote The Vision of Sin, Babbage read it. After doing so, it is said he wrote the following extraordinary letter to the poet.

In your otherwise beautiful poem, there is a verse which reads

Every moment dies a man
Every moment one is born.

It must be manifest that, were this true, the population of the world would be at a standstill. In truth the rate of birth is slightly in excess of that of death. I would suggest that in the next edition of your poem you have it read

Every moment dies a man

Every moment $1\frac{1}{16}$ is born.

Strictly speaking this is not correct. The actual figure is a decimal so long that I cannot get it in the line, but I believe the 1 1/16 will be sufficiently accurate for poetry. I am, etc.

World of Mathematics, Vol.3 p.1487.)

CAN ANYONE APPRECIATE MATHEMATICAL BEAUTY ?

It is natural to wonder whether only mathematicians can appreciate mathematical beauty, or whether anybody can. Hardy's answer, in A Mathematician's Apology, is as follows -

APPRECIATION OF MATHEMATICAL BEAUTY NOT RARE

It would be difficult to find an educated man quite insensitive to the aesthetic appeal of mathematics..... The fact is that there are few more "popular" subjects than mathematics. Most people have some appreciation of mathematics, just as most people have xxxx can enjoy a pleasant tune ; and there are probably more people really interested in mathematics than in music.... There are masses of chess-players in every civilized country - in Russia almost the whole educated population; and every chess player can recognize and appreciate a "beautiful" game or problem . Yet a chess problem is simply an exercise in pure mathematics (a game not entirely since psychology plays a part) and everyone who calls a problem "beautiful" is applauding mathematical beauty, even if it is beauty of a comparatively lowly kind. Chess problems are the hymn tunes of mathematics.

(He goes on to talk about bridge problems and the puzzles in newspapers ; speaking of puzzles he says -) What the public wants is a little intellectual "kick", and nothing else has quite the "kick" of mathematics.

Hardy. A Mathematician's Apology.

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Some sources for the lives of mathematicians, and related topics.

1. E.T.Bell. Men of Mathematics.
2. Tobias Dantzig; Number, The Language of Science.
3. W.W.R.Ball. A Short History of Mathematics.
4. J.Hadamard. The Psychology of Invention in the Mathematical Field.
5. Felix Klein. Die Entwicklung der Mathematik im XIX. Jahrhundert.
6. Preface to Riemann's Collected Works (Dover) for life of Riemann.